The Electric Field

Definition of the electric field:

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_0}$$

SI unit: N/C

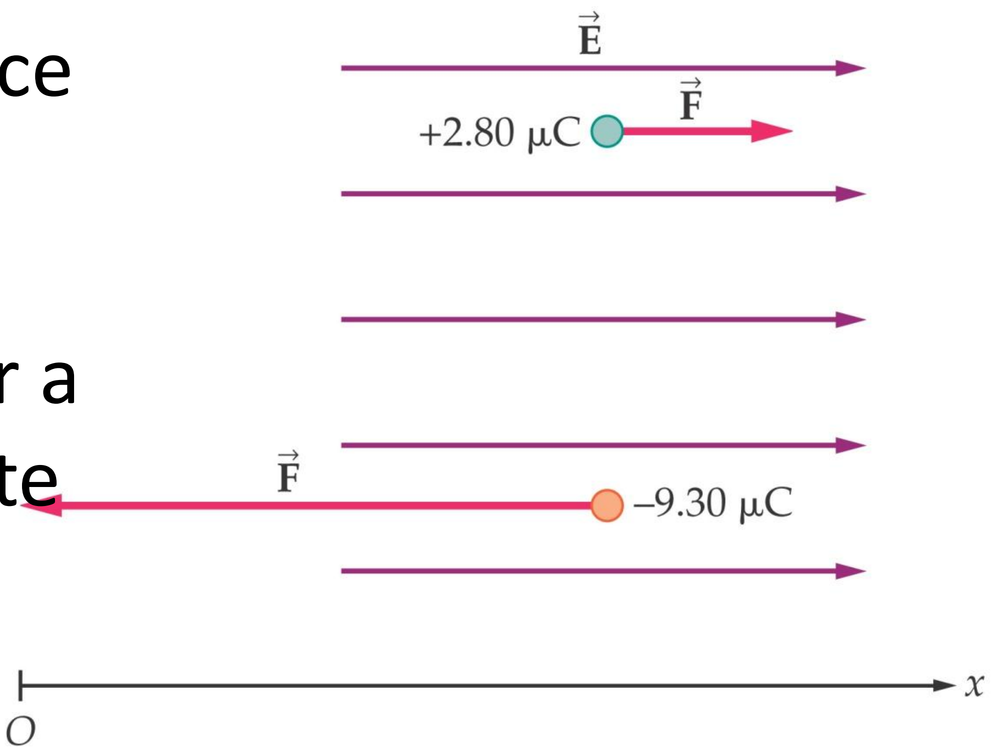
Here, q_0 is a "test charge" – it serves to allow the electric force to be measured, but is not large enough to create a significant force on any other charges.

The Electric Field

If we know the electric field, we can calculate the force on any charge:

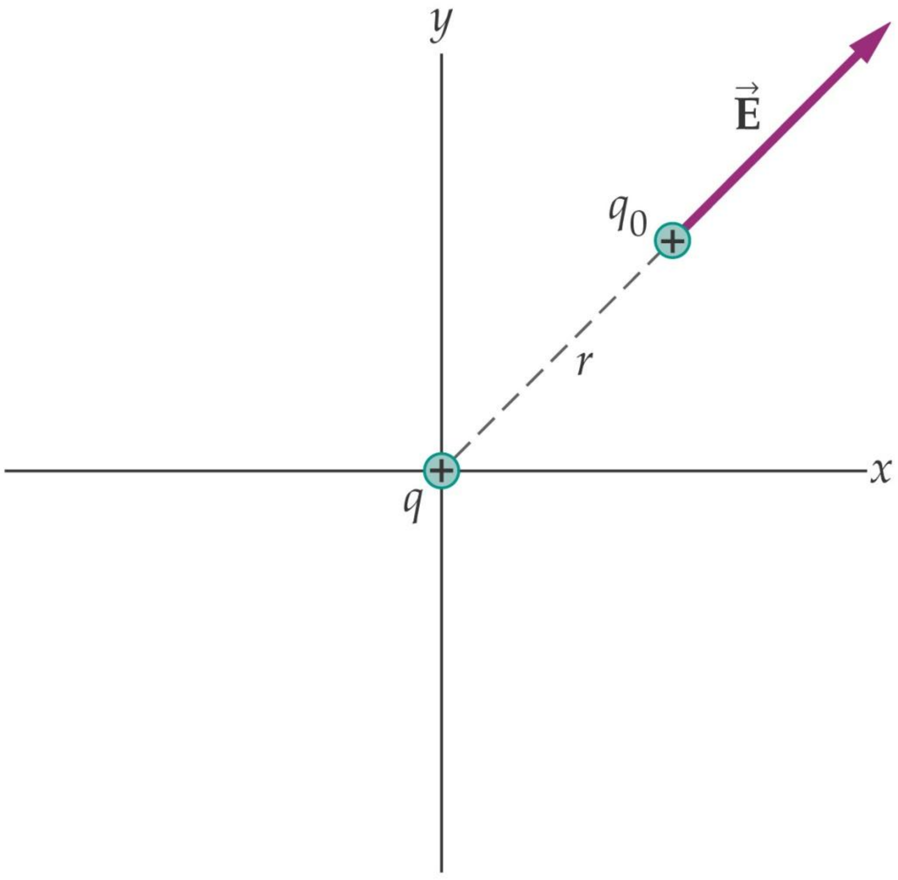
$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

The direction of the force depends on the sign of the charge – in the direction of the field for a positive charge, opposite to it for a negative one.



The Electric Field

The electric field of a point charge points radially away from a positive charge and towards a negative one.



Electric Field Lines

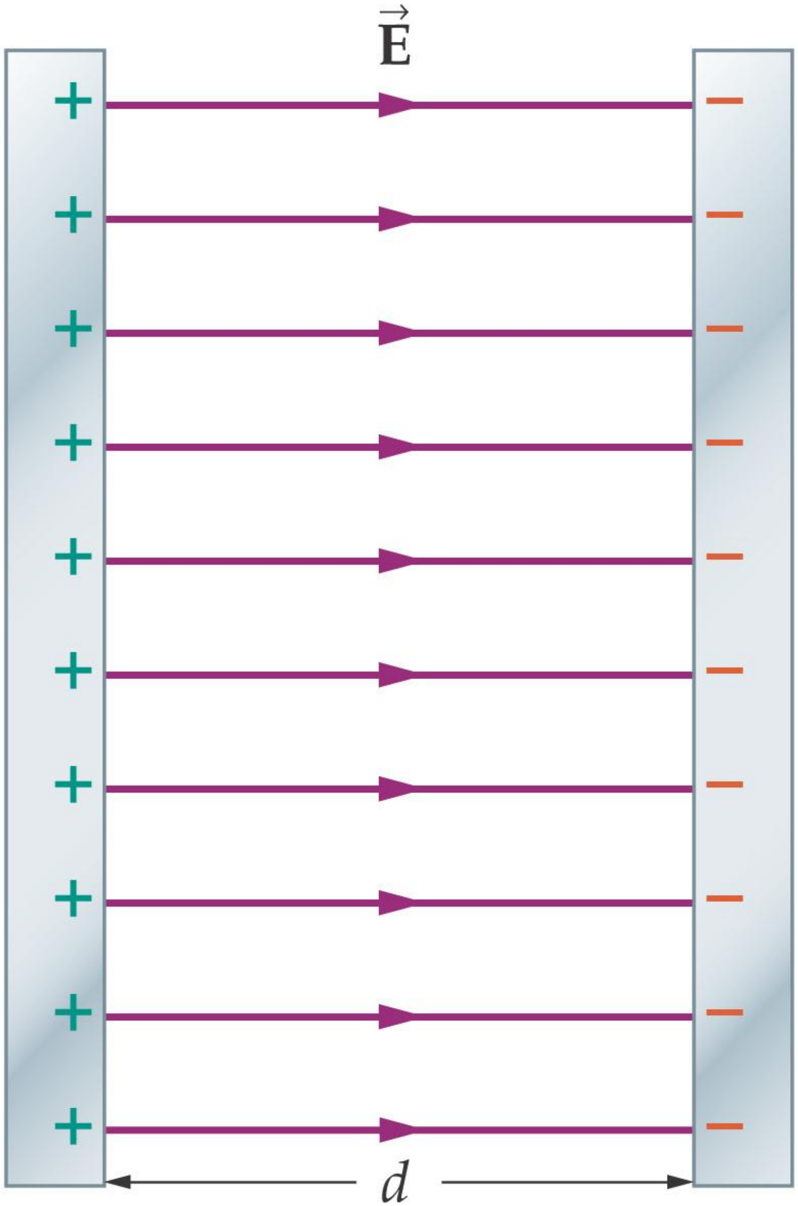
Electric field lines are a convenient way of visualizing the electric field.

Electric field lines:

1. Point in the direction of the field vector at every point
2. Start at positive charges or infinity
3. End at negative charges or infinity
4. Are more dense where the field is stronger

Electric Field Lines

A parallel-plate capacitor consists of two conducting plates with equal and opposite charges. Here is the electric field:



Coulomb's Law for the Field

Coulomb's law for the force on *q* due to *Q*:

$$\vec{F} = k\frac{qQ}{r^2} \quad = q\vec{E}$$

Coulomb's law for the field *E* due to *Q*:

$$\boxed{\vec{E} = k\,\frac{Q}{r^2}}$$

Example 1

What is the electric field strength at a distance of 10 cm from a charge of 2μC?

$$E = \frac{kQ}{r^2} = \frac{9 \times 10^{9} \times 2 \times 10^{-6}}{(10 \times 10^{-2})^2}$$

$$= \frac{18 \times 10^{3}}{10^{-2}} = 1.8 \times 10^{6}\ N/C$$

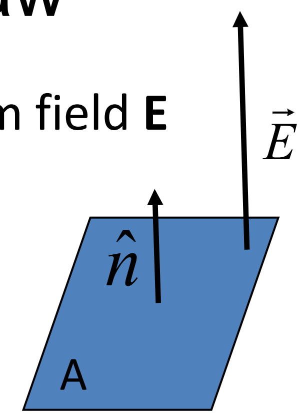
So a one-coulomb charge placed there would feel a force of 1800,000 Newton.

Gauss's Law

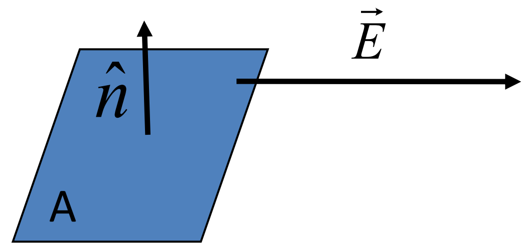
- Gauss's law makes it possible to find the electric field easily in highly symmetric situations.
- Drawing electric field lines around charges leads us to Gauss' Law
- The idea is to draw a closed surface like a balloon around any charge distribution, then some field line will exit through the surface and some will enter or renter. If we count those that leave as positive and those that enter as negative, then the net number leaving will give a measure of the net positive charge inside.

Electric lines of flux and Gauss's Law

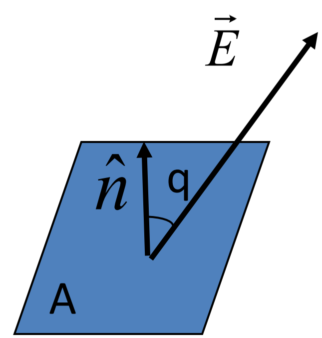
- The flux ϕ through a plane surface of area **A** due to a uniform field **E** is a simple product:
  ϕ = E A where E is normal to the area A .

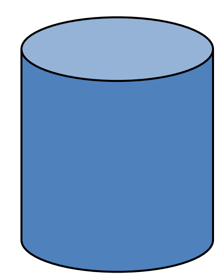


- $\phi = E_n A = 0 \text{ x } A = 0$ because the normal component of E is 0



- $\phi = E_n A = E \cos q\ A$





Approximate Flux

$$\Phi = \sum \vec{E} \bullet \Delta\vec{A}$$

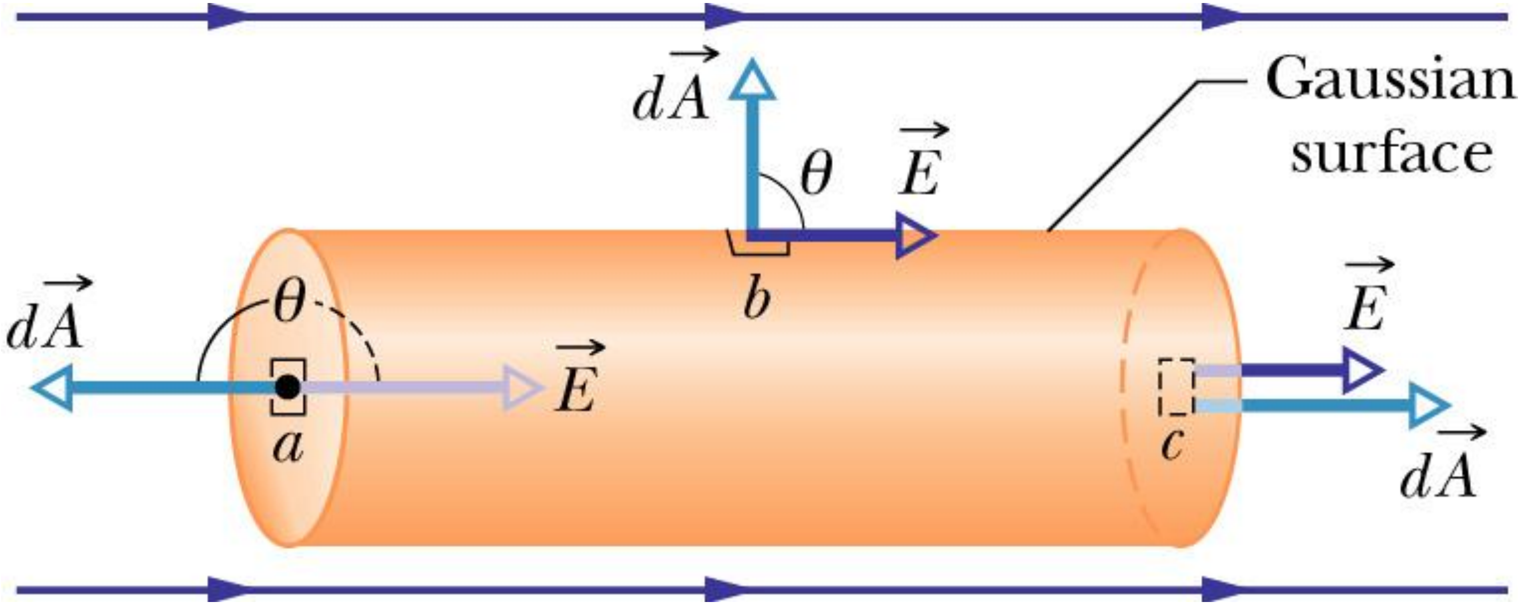
Exact Flux

$$\Phi = \oint \vec{E} \bullet d\vec{A}$$

$$d\vec{A} = \hat{n}\, dA$$

Circle means you integrate over a closed surface.

Find the electric flux through a cylindrical surface in a uniform electric field **E**



$$\Phi = \oint \vec{E} \bullet d\vec{A} \qquad = \oint E \cos\theta dA \qquad d\vec{A} = \hat{n} dA$$

a. $\Phi = \int E \cos 180 dA = -\int E dA = -E\pi R^2$

b. $\Phi = \int E \cos 90 dA = 0$

c. $\Phi = \int E \cos 0 dA = \int E dA = E\pi R^2$

Flux from a. + b. + c. = 0

Electric lines of flux and Derivation of Gauss' Law using Coulombs law

- Consider a sphere drawn around a positive point charge. Evaluate the net flux through the closed surface.

Net Flux = $\Phi = \oint \vec{E} \bullet d\vec{A} = \oint E\cos\theta dA = \oint EdA$

E || n
Cos 0 = 1

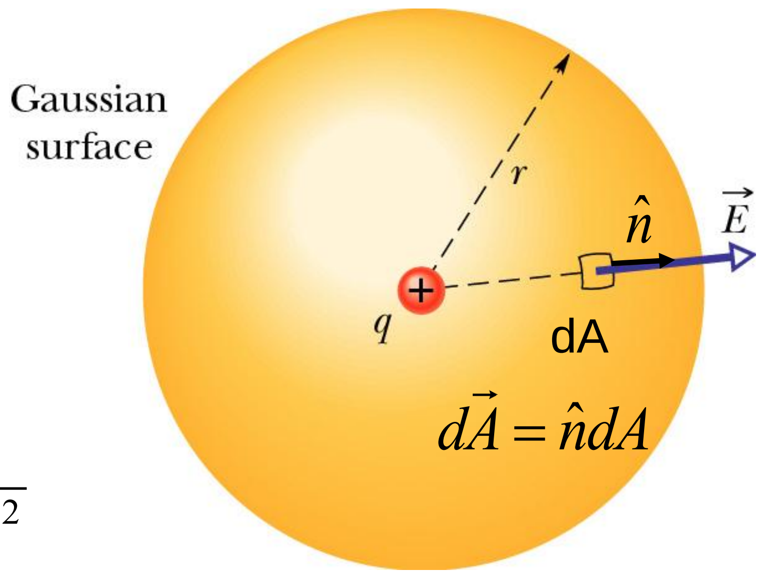
For a Point charge **E=kq/r²**

$$\Phi = \oint EdA = \oint kq/r^2 dA$$

$$\Phi = kq/r^2 \oint dA = kq/r^2(4\pi r^2)$$

$$\Phi = 4\pi kq$$

$$4\pi k = 1/\varepsilon_0 \text{ where } \varepsilon_0 = 8.85\text{x}10^{-12}\frac{C^2}{Nm^2}$$



$$\boxed{\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}}$$ Gauss' Law

Gauss' Law

$$\Phi_{net} = q_{enc} / \varepsilon_0$$

This result can be extended to any shape surface with any number of point charges inside and outside the surface as long as we evaluate the net flux through it.